The warranty policy under fuzzy environment

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Abstract

Purpose – The purpose of this paper is to propose a model that captures the fuzzy events is proposed to find the optimal periods of warranty policies. The model considers repair and replacement actions in the warranty period.

Design/methodology/approach – The study transforms the reliability of a traditional set to a fuzzy reliability set that models a problem. The optimality of the model is explored with classical optimal theory. Also, a numerical example is presented to describe how to find an optimal warranty policy.

Findings – The study proves that the optimality of a warranty model can be used to find the optimal warranty policy in a fuzzy environment.

Originality/value – The model is useful for firms in deciding what the maintenance strategy and warranty period should be in a fuzzy environment.

Keywords Warranties, Failure (mechanical), Reliability management, Maintenance, Mathematical modelling

Paper type Research paper

1. Introduction

Modern manufacturing firms face fierce competition because of rapidly changing technologies, nearly identical products, and better educated and more demanding customers (Murthy and Djamaludin, 2002). In promoting products, firms are hard pressed to provide more attractive offers to buyers. For durable products, especially when the competing brands are nearly identical, buyers typically choose a particular product on the basis of its price, perceived quality and reliability, features, and financing provided by the manufacturer. Lele (1983) and Lele and Karmarkar (1983) note that the availability of warranty parts and post-sale maintenance service have added importance in product choice. Furthermore, for a durable product, the greater the innovation and sophistication of the processes, the more concern over reliability that customers will have. Therefore, in marketing durable products, the warranty is an important instrument to signal higher product quality and greater assurance to customers. Warranty is defined as a contractual obligation of a manufacturer in selling a product to ensure that the product functions properly during the warranty period

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(Blischke and Murthy, 1994). Other research has shown that different competitive advantages can be attained from warranty policies (Huang and Zhuo, 2004; Iskandar *et al.*, 2005). However, servicing warranty involves additional costs to the manufacturer – costs relating to product reliability and warranty terms.

In exploring the optimality of a warranty policy, many issues are proposed, such as the renewed period of warranty (Chukova and Hayakawa, 2004; Mitra and Patankar, 1997; Thomas, 1989), the cost structure for replacement (Barlow and Hunter, 1960), repair (Tilquin and Cléroux, 1975), periodic replacement of shock (Boland and Proschan, 1982), optimal time for repair (Sheu, 1993), optimal time of repair or replacement (Jhang and Sheu, 1995), minor and catastrophic failure (Sheu, 1993), optimal maintenance (Juang and Anderson, 2004; Wang and Sheu, 2003; Bai and Pham, 2004), and association between price, quality, and warranty period (Lin and Shue, 2005; Lin *et al.*, 2001; Kotler, 1976; Peterson, 1970). Some of these studies assume that the failure process obeys a non-homogeneous procession process (Nakagawa and Kowada, 1983; Savits, 1988; Juang and Anderson, 2004). Therefore, the optimality of the warranty policy should be considered in terms of the relationship between the price and the warranty policy (Kotler, 1976; Peterson, 1970).

Since randomness is not merely an aspect of uncertainty in many fields of application, the fuzziness of the environment cannot be neglected in modeling an observed process. It is difficult to capture lifetime data on reliability in polluted and imprecise situations, especially for new and durable products, non-mass products, and short product development times. Usually, there is no comparative reliability information available, the lifetime data trend to be based on subjective evaluation or rough estimate (Huang *et al.*, 2006). To deal with the problems above, the modeling of reliability distribution has to be based on the fuzziness of lifetime data. Many studies also demonstrate that the fuzzy theory is suitable for modeling the reliability property of a product (e.g. Cai *et al.*, 1991; Cheng, 1996; Huang, 1995; Onisawa and Kacprzyk, 1995; Lin and Shue, 2005). Hence, it is desirable to incorporate the fuzzy theory into the failure distribution for the warranty model. With this in mind, one study explores the product repair and replacement strategies by evaluating the relevant warranty cost factors to a fuzzy reliability model.

The remainder of this paper is presented as follows. Section 2 presents the model, including how it is formulated. Then, in section 3, a numerical example is presented to demonstrate the solution process. The final section presents conclusions and suggestions for future studies.

2. Model formulation

2.1 Problem characterization

Suppose a system has two types of failure when the system fails at age t during the time interval $(0, w]$:

(1) type I failure (minor failure) occurs with probability $q_i(t)$ and minimal repair will be taken to recover the system; and

(2) type II failure (catastrophic failure) occurs with probability $p_i(t) = 1 - q_i(t)$ and replacement action will be taken to recover the system.

The combination of the product price function and warranty cost function to formulate the revenue function with the warranty period $(0, w]$ is stated as:

$$\max \pi = P(w) - C(w), \tag{1}$$

subject to $P(w) > 0$ and $C(w) > 0$, where π is the unit profit, $P(w)$ is the product price with the warranty period $(0, w]$, and $C(w)$ is the warranty cost with the warranty period $(0, w]$.

2.1 Parameter setting

Let $E_i(c_{\mathrm{I}})$ and $E_i(c_{\mathrm{II}})$ denote the expected repair cost for the ith type I failure and the expected replacement cost for the ith type II failure, respectively, for part i, $i = 1, \ldots, n$:

$$\max \pi = P(w) - \sum_{i=1}^{n} [E_i(c_{\mathrm{I}}) + E_i(c_{\mathrm{II}})], \tag{2}$$

subject to $w, n > 0$. If type II failure takes place for a part before time t during the warranty period $(0, w]$, then replacement action should be taken. In addition, the warranty coverage is renewed (i.e. the renewed warranty period remains the same as if the initial one has been provided again). For type I failure, only the repair action is conducted.

In the real world, it may be difficult to collect enough data to derive the probability distribution of a certain event. Therefore, this study will model the reliability event with fuzzy theory.

Let:

$$\tilde{R}(t) = P(T \gtrsim t) = \int_t^{\infty} \mu(x) f(x)\, \mathrm{d}x \; 0 \leq t \leq x < \infty, \tag{3}$$

where T, $f(x)$ and $\mu(x)$ denote the life cycle, probability density function, and membership function, respectively. Also, let $x(\alpha)$ correspond to the value of the α-cut; then the corresponding reliability function can be stated as:

$$\tilde{R}_\alpha(t) = \int_t^{x(\alpha)} f(x)\, \mathrm{d}x. \tag{4}$$

Moreover, the probability function of failure is given by:

$$\tilde{F}_\alpha(t) = 1 - \tilde{R}_\alpha(t) = 1 - \int_t^{x(\alpha)} f(x)\, \mathrm{d}x, \; 0 \leq t \leq x < \infty. \tag{5}$$

With t as a constant, we obtain:

$$\mathrm{d}\tilde{F}_\alpha(t) = -f[x(\alpha)] \times x'_\alpha(\alpha)\, \mathrm{d}\alpha. \tag{6}$$

Similarly, by letting α be a constant, we obtain:

$$\mathrm{d}\tilde{F}_\alpha(t) = [f(t) - f[x(t)] \times x'_t(t)]\, \mathrm{d}t. \tag{7}$$

The expected cost functions of $E_i(c_{\mathrm{I}})$ and $E_i(c_{\mathrm{II}})$ can be given as:

$$E_i(c_{\mathrm{I}}) = \int_0^w \int_0^t h_i(y) q_i(y) r_i(y)\,\mathrm{d}y\,\mathrm{d}\tilde{F}_{\alpha i}, \tag{8}$$

and

$$E_i(c_{\mathrm{II}}) = c_{\mathrm{r}i} \times \tilde{F}_{\alpha i}(w), \tag{9}$$

respectively, where $h_i(y)$ is the minimum expected cost of minor failure for item i at age y during the period $(0, w]$, $q_i(y)$ is the probability of minor failure at age y during the period $(0, w]$, $r_i(y)$ is the failure rate of item i at age y during the period $(0, w]$, $c_{\mathrm{r}i}$ is the the replacement cost of item i, and $\tilde{F}_{\alpha_i}(w)$ is the fuzzy cumulative distribution function that uses the α-cut approach.

Substituting equations (8) and (9) into equation (2), we obtain equation (10), which describes the profit function:

$$\pi = P(w) - \sum_{i=1}^{n} \left[\int_0^w \int_0^t h_i(y) q_i(y) r_i(y)\,\mathrm{d}y\,\mathrm{d}\tilde{F}_{\alpha i} + c_{\mathrm{r}i} \times \tilde{F}_{\alpha i}(w) \right]. \tag{10}$$

The optimality of the profit function will be explored and its conditions will be derived in the following theorems.

Theorem 1. The profit function π will increase when α increases, assuming that the profit function is continuous and differentiable, where $0 < \alpha < 1$.

Proof. Combining equation (7) with equation (10), we have:

$$\pi(\alpha, t) = P(w) - \sum_{i=1}^{n} \left[\int_0^w \int_0^t h_i(y) q_i(y) r_i(y)\,\mathrm{d}y \times [-f_i(x(\alpha)) \times x'_{\alpha i}(\alpha)]\,\mathrm{d}\alpha + c_{ri} \times \tilde{F}_{\alpha i}(w) \right],$$

$$\frac{\partial \pi(\alpha, t)}{\partial \alpha} = P'_\alpha(w) - \sum_{i=1}^{n} \left[\int_0^t h_i(y) q_i(y) r_i(y)\,\mathrm{d}y \times [-f_i(x_i(\alpha)) \times x'_{\alpha_i}(\alpha)] + c_{\mathrm{r}i} \times [-f_i(x_i(\alpha)) \times x'_{\alpha_i}(\alpha)] \right]$$

$$= P'_\alpha(w) + \sum_{i=1}^{n} \left[\int_0^t h_i(y) q_i(y) r_i(y)\,\mathrm{d}y + c_{\mathrm{r}i} \right] \times [f(x_i(\alpha)) \times x'_{\alpha_i}(\alpha)].$$

Since $P(w)$ is a real function, taking the derivative in terms of α and letting $P'(w) = 0$ results in:

$$\sum_{i=1}^{n} \left[\int_0^t h_i(y) q_i(y) r_i(y)\,\mathrm{d}y + c_{\mathrm{r}i} \right] \times [f(x_i(\alpha)) \times x'_{\alpha_i}(\alpha)]. \tag{11}$$

The cost of minimal and catastrophic failure will be greater than zero. Additionally, the density function of failure $f[x_i(\alpha)]$ is positive, and the value of the independent function

$x_{\alpha_i}(\alpha)$ will increase progressively when α increases. Therefore, $[\partial \pi(\alpha, t)]/(\partial \alpha) > 0$, i.e. the profit function π will increase when α increases.

Theorem 1 shows that the unit profit will increase when the α-cut, which belongs to the membership function, increases until the value of α approaches 1. It means that the more information we obtain for the problem, the more confidence the decision maker(s) own(s), and the greater the profit will be attained.

Theorem 2. t is a critical value of the profit function if $[\partial \pi(\alpha, t)]/(\partial t) = 0$, where t is a variable of the profit function and α is a constant.

Proof. Substituting equation (7) into equation (10) results in:

$$\pi = P(w) - \sum_{i=1}^{n} \left[\int_0^w \int_0^t h_i(y) q_i(y) r_i(y)\, \mathrm{d}y \times [f_i(t) - f_i[x(t)] \times x_i'(t)]\, \mathrm{d}t + c_{\mathrm{r}i} \times \tilde{F}_{\alpha_i}(w) \right]. \quad (12)$$

The partial derivative of π with respect to t is:

$$\frac{\partial \pi(t, \alpha)}{\partial t} = P_t'(w) - \sum_{i=1}^{n} \left[\int_0^t h_i(y) q_i(y) r_i(y)\, \mathrm{d}y \times [f_i(t) - f_i(x(\alpha)) \times x_{t_i}'(\alpha)] + c_{\mathrm{r}i} \times [f_i(t) - f_i(x(\alpha)) \times x_{t_i}'(\alpha)] \right]. \quad (13)$$

Let $[\partial \pi(t, \alpha)]/(\partial t) = 0$. We then have:

$$P_t'(w) = \sum_{i=1}^{n} \left[\int_0^t h_i(y) q_i(y) r_i(y)\, \mathrm{d}y + c_{\mathrm{r}i} \right] \times [f_i(t) - f_i(x(\alpha)) \times x_{t_i}'(\alpha)]. \quad (14)$$

Note that the warranty price $P(w)$ depends on the warranty period t. Since the warranty period and product price are positively correlated, $P_t'(w)$ will be a positive number. In addition, $\int_0^t h_i(y) q_i(y) r_i(y)\, \mathrm{d}y + c_{ri}$ represents the cost of repair and the replacement for part i, which are greater than zero. The first-order derivative in terms of time t can be stated by $f_i(t) - f_i[x(\alpha)] \times x_{t_i}'(\alpha)$, which is greater than zero. Therefore, the values of the two sides of equation (14) are of the same sign and greater than zero, and Theorem 2 is proved.

Combining the fuzzy-reliability function (equation ((7)) with equation (10), and taking the derivation with respect to $\pi(\alpha, t)$ in terms of t, and letting it equal zero, we can determine the critical number of warranty period t.

Theorem 3. There exists a maximum value for the profit function in terms of t for a fixed α.

Proof. Taking the second derivation of profit function in terms of t:

$$\frac{\partial^2 \pi(\alpha,t)}{\partial t^2} = P_t''(w) - \sum_{i=1}^{n} \left[h_i(t)q_i(t)r_i(t) \times [f_i(t) - f_i(x(\alpha)) \times x_{t_i}''(\alpha)] \right.$$

$$+ \int_0^t h_i(y)q_i(y)r_i(y)\,\mathrm{d}y \times [f_{t_i}'(t) - f_{t_i}'(x(\alpha)) \times x_{t_i}'(\alpha) - f_i(x(\alpha))$$

$$\left. \times x_{t_i}''(\alpha)] + c_{ri} \times [f_{t_i}'(t) - f_{t_i}'(x(\alpha)) \times x_{t_i}'(\alpha) - f_i(x(\alpha)) \times x_{t_i}''(\alpha)] \right. . \qquad (15)$$

Suppose the price function is linear in terms of t. Then, $P_t''(w) = 0$. For a triangular membership function, $x_{t_i}''(\alpha)$ is zero. Hence, $f_i[x(\alpha)] \times x_{t_i}''(\alpha) = 0$. Furthermore, $f_{t_i}'(t) - f_{t_i}'[x(\alpha)] \times x_{t_i}'(\alpha) > 0$, and thus $[\partial^2 \pi(\alpha,t)]/(\partial t^2) < 0$.

Therefore, according to Theorems 2 and 3, we can solve the function $[\partial^2 \pi(\alpha,t)]/(\partial t^2) = 0$ to obtain the optimal solutions of t and the profit. In the following section, we will demonstrate the solution procedure based on the case that the failure function obeys a Weber distribution.

3. A numerical example

A numerical example is presented here. According to the fuzzy theory, the membership function can be depicted as in Figure 1 (in Figure 1, t_1 is th most pessimistic failure time, and t_2 is the most optimistic failure time).

Suppose the PDF of failure time t follows Weber distribution, which is stated as:

$$f(x) = \frac{\beta}{\theta}\left(\frac{x}{\theta}\right)^{\beta-1} \mathrm{e}^{-\left(\frac{x}{\theta}\right)^{\beta}},\ x > 0,\ \beta > 1,\ \theta > 0, \qquad (16)$$

where x is the time interval between failures, β is the shape parameter, θ is the life span of character (greater than zero), and the membership function can be expressed by:

$$\mu_R(x(\alpha)) = \begin{cases} 0, & x(\alpha) \le t \\ \frac{1}{1+(x(\alpha)-t)^{-\beta}}, & x(\alpha) > t,\ \beta > 0 \end{cases}, \qquad (17)$$

where $x(\alpha)$ can be expressed by:

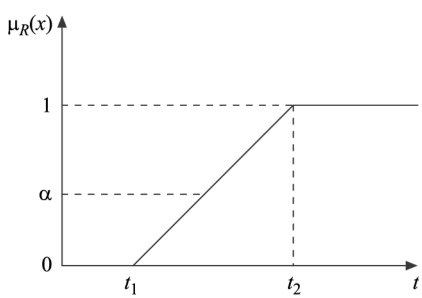

Figure 1.
Membership function

$$\begin{cases} x(\alpha) \le t, & \alpha = 0 \\ x(\alpha) = t + \left(\frac{\alpha}{1-\alpha}\right)^{1/\beta}, & 0 < \alpha < 1 \end{cases}, \tag{18}$$

where $\alpha \neq 1$.

The fuzzy reliability functions can be expressed by:

$$\tilde{R}_\alpha(t) = \int_t^{t+\left(\frac{\alpha}{1-\alpha}\right)^{1/\beta}} \frac{\beta}{\theta}\left(\frac{x}{\theta}\right)^{\beta-1} e^{-\left(\frac{x}{\theta}\right)^\beta} dx = e^{-\left(\frac{t}{\theta}\right)^\beta} - e^{-\left(\frac{t+\left(\frac{\alpha}{1-\alpha}\right)^{1/\beta}}{\theta}\right)^\beta}. \tag{19}$$

Remark. When $\alpha = 0$:

$$\tilde{R}_\alpha(t) = \int_t^t \frac{\beta}{\theta}\left(\frac{x}{\theta}\right)^{\beta-1} e^{-\left(\frac{x}{\theta}\right)^\beta} dx = 0.$$

When $\alpha \to 1$:

$$\lim_{\alpha \to 1^-} \tilde{R}_\alpha(t) = e^{-\left(\frac{t}{\theta}\right)^\beta}.$$

The cost function of a catastrophic failure can be expressed by:

$$E_i(c_{\mathrm{II}}) = c_{\mathrm{r}i} \times \tilde{F}_{\alpha_i}(t) = c_{\mathrm{r}i} \times \left\{1 - e^{-\left(\frac{t}{\theta_i}\right)^{\beta_i}} + e^{-\left(\frac{t+\left(\frac{\alpha}{1-\alpha}\right)^{1/\beta_i}}{\theta_i}\right)^{\beta_i}}\right\}. \tag{20}$$

Remark. When $\alpha = 0$:

$$E_i(c_{\mathrm{II}}) = c_{\mathrm{r}i} \times \tilde{F}_{\alpha_i}(t) = c_{\mathrm{r}i}.$$

When $\alpha \to 1$:

$$E_i(c_{\mathrm{II}}) = c_{\mathrm{r}i} \times \tilde{F}_{\alpha_i}(t) = c_{\mathrm{r}i} \times \left[1 - e^{-\left(\frac{t}{\theta_i}\right)^{\beta_i}}\right].$$

The cost function of a minimal failure is derived and the implication is demonstrated in the following paragraphs.

$$\begin{aligned} E(c_{\mathrm{I}}) &= \int_t^{x(\alpha)} \int_0^t h(y)q(y)r(y)\, dy\, d\tilde{F}_\alpha(t) \\ &= \int_0^{x(\alpha)} \int_0^t \left(\frac{1}{q(y)} \int_0^{\delta(y)c_\infty} xnor(x)\, dx + c\right) \times q(y) \times r(y)\, dy\, d\tilde{F}_\alpha(t). \end{aligned}$$

Since

$$E[C(y)] = \frac{1}{q(y)} \int_0^{\delta(y)c_\infty} xnor(x)\, dx = \frac{\mu}{q(y)},$$

and:

$$r(y) = \frac{f(y)}{R(y)} = \frac{\frac{\beta}{\theta}\left(\frac{y}{\theta}\right)^{\beta-1} e^{-\left(\frac{y}{\theta}\right)^{\beta}}}{e^{-\left(\frac{y}{\theta}\right)^{\beta}}} = \frac{\beta}{\theta}\left(\frac{y}{\theta}\right)^{\beta-1},$$

we have:

$$E(c_{\mathrm{I}}) = \int_t^{x(\alpha)} \int_0^t [\mu + c(y)] \times \frac{\beta}{\theta}\left(\frac{y}{\theta}\right)^{\beta-1} \mathrm{d}y\, \mathrm{d}\tilde{F}_\alpha = \int_t^{x(\alpha)} (\mu + c)\left(\frac{t}{\theta}\right)^{\beta} \mathrm{d}\tilde{F}_\alpha. \quad (21)$$

Taking $x(\alpha) = t + [\alpha/(1-\alpha)]^{1/\beta}$ into equation (21), we get:

$$\begin{aligned} E(c_I) &= \int_w^{w+\left(\frac{\alpha}{1-\alpha}\right)^{1/\beta}} (\mu + c)\left(\frac{t}{\theta}\right)^{\beta} \mathrm{d}\tilde{F}_\alpha \\ &= (\mu + c)\left\{1 + e^{-\frac{w+\left(\frac{\alpha}{1-\alpha}\right)^{1/\beta}}{\theta}}\left[-1 - \left(\frac{w + \left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{\beta}}}{\theta}\right)^{\beta_i}\right]\right\}. \end{aligned} \quad (22)$$

This reveals that the cost of minimal repair and catastrophic failure will decrease.

Remark. When $\alpha = 0$:

$$E_i(c_{\mathrm{I}}) = \int_0^{x(\alpha)} \int_0^t h_i(y) q_i(y) r_i(y)\, \mathrm{d}y\, \mathrm{d}\tilde{F}_{\alpha_i}(t) = 0\ (\because \mathrm{d}\tilde{F}_{(\alpha=0)}(t) = 0).$$

When $\alpha \rightarrow 1$:

$$E(c_{\mathrm{I}}) = \int_t^{x(\alpha)} \int_0^t h(y) q(y) r(y)\, \mathrm{d}y\, \mathrm{d}\tilde{F}_\alpha = \mu + c.$$

We explore the policy of the warranty period w in terms of α value as follows.

The profit function is expressed by:

$$\begin{aligned} \pi = P(w) - &\left\{1 + e^{-\frac{w+\left(\frac{\alpha}{1-\alpha}\right)^{1/\beta_i}}{\theta_i}}\left[-1 - \left(\frac{w + \left(\frac{\alpha}{1-\alpha}\right)^{1/\beta_i}}{\theta_i}\right)^{\beta_i}\right]\right\} + c_{ri} \\ &\times \left[1 - e^{-(w/\theta_i)^{\beta_i}} + e^{-\left(\frac{w+\left(\frac{\alpha}{1-\alpha}\right)^{1/\beta_i}}{\theta_i}\right)^{\beta_i}}\right]. \end{aligned} \quad (23)$$

Remark. When $\alpha = 0$, the profit function is expressed as:

$$\pi = P(w) - \sum_{i=1}^{n} c_{ri}.$$

This implies that for any failure in the warranty period w, we adopt a replacement strategy instead of minimal repair in order to minimize costs.

When $\alpha \rightarrow 1$, the profit function is expressed as:

$$\pi = P(w) - \sum_{i=1}^{n} (\mu_i + c_i) + c_{ri}\left(1 - e^{-\left(\frac{w}{\theta_i}\right)^{\beta_i}}\right).$$

For a specific α, we can find the optimal value of the warranty period based on the proposed theorems. Suppose $P(w) = 250w + 50$. The product consists of three items for which the replacement costs of items are $c_{r1} = 200$, $c_{r2} = 250$, $c_{r3} = 280$, respectively, and the costs of minimal repair are $\mu_1 + c_1 = 100$, $\mu_2 + c_2 = 125$, $\mu_3 + c_3 = 140$, respectively. The parameter values for the distribution are $\theta_1 = 2$, $\theta_2 = 4$, $\theta_3 = 5$, $\beta_1 = 1$, $\beta_2 = 2$, and $\beta_3 = 2$. Table I shows the profit for the different warranty periods, and the optimal solutions can be found for $\alpha = 0.5$ and $\alpha = 0.8$.

Table I reveals that when $\alpha = 0.5$ the optimal warranty period is $w = 4$, while when $\alpha = 0.8$, the optimal warranty period is $w = 3$. The increase in cost for minor repair is greater than the decrease in cost of replacement when α increases, resulting in the optimal warranty period $w = 3$ to attain the maximum profit in this case.

4. Conclusions

Nowadays, a warranty is necessary to enhance a product's sales. An appropriate warranty period and the cost of this warranty should be determined based on the product's failure rate. This study proposes a model to create a formula for maintenance and replacement strategies in a fuzzy environment. Although the fuzzy reliability model adopted here is more complex than the classical model, the evidence shows that the classical model is inadequate for a fierce competitive environment. The reasons for this are as follows. It is inherent in reliability analysis to collect a relatively large amount of lifetime data because the classical reliability estimation is typically based on precise lifetime data. However, with new industrial technologies, demanding customers and shorter product development cycles, the lifetime of products has become contradictory. It is time-consuming, expensive, and sometime impossible to obtain enough exact observations to fit the lifetime distribution (Kenarangui, 1991; Park and Kim, 1990; Tanaka *et al.*, 1983). With few available data points, it is difficult to estimate the lifetime distribution parameters using conventional reliability analysis methods. Hence, to enhance the success of marketing a new product, fuzzy theory or other statistical theories (Huang *et al.*, 2006) should be implemented, by capturing the

w	$P(w)$	α	$\sum_{i=1}^{3} E_i(c_{\mathrm{I}})$	$\sum_{i=1}^{3} E_i(c_{\mathrm{II}})$	π
1	300	0.5	28.43	611.70	−340.13
2	550	0.5	55.92	605.54	−111.46
3	800	0.5	104.82	614.26	80.92
4	1,050	0.5	217.03	635.15	197.82
5	1,300	0.5	509.01	660.82	130.16
1	300	0.8	82.97	459.03	−242.00
2	550	0.8	125.50	472.69	−48.19
3	800	0.8	232.17	509.03	58.80
4	1,050	0.8	519.77	559.67	−29.44

Table I. The profit for different warranty periods (Weber distribution and $0 < \alpha < 1$)

experience, subject judgment and available lifetime data to fit the reliability distribution in a faster way. However, in order to formulate a warranty model, the firm still needs to collect relevant data for all parameters of the model. The cost and time necessary to collect such data must be estimated before determining the warranty models.

This study is useful for firms in deciding what the maintenance strategy and warranty period should be. It also allows for an extended warranty price to be derived if the function of cost elasticity is available. However, several issues are neglected in this study. For example, parallel connection among the parts and the connection between sales and warranty price are not examined. Moreover, the fuzzy properties of cost and other relevant factors in the model are worthwhile topics to be examined in the future.

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